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Electron–hole correlation in fractional quantum Hall systems

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Abstract. The 2D system of electrons confined to the lowest Landau level is described using a representation with a density matrix depending both on electron and hole coordinates. Condensation of the electron system into a fractional quantum Hall state is associated with clustering of particle and hole coordinates. The correlation of particle and hole coordinates is studied and ground-state wave functions are derived for $\nu = p/(2p \pm 1)$. These wave functions prove to be accurate for the cases studied, i.e. for $\nu = 2/3$ and $3/5$, and are identical to *pair* wave functions (Morf R, d'Ambrumenil N and Halperin B I 1986 *Phys. Rev.* B **34** 3037) for $\nu = 2/3$ and $2/5$.

The theory of Laughlin [1] provides a good microscopic description of the $\nu = 1/m$ incompressible quantum Hall states, with m odd. On the basis of this description, it has been proposed that one could identify fractional quantum Hall states by the occurrence of the binding of flux quanta to electrons [2]. An explicit formulation for such a binding is provided by the composite-fermion approach [3, 4], which makes use of a singular gauge transformation to map the system of electrons in the presence of a magnetic field onto a system of composite fermions, which are objects each composed of an electron bound to an even number of flux quanta.

In this article, an alternative formulation is presented for treating the binding of flux quanta to electrons. This approach makes use of a particular representation of the finite-temperature density matrix for the electron system confined to the lowest Landau level, and consists in the evaluation of matrix elements of the density matrix operator connecting states parametrized by electron coordinates and states parametrized by the coordinates of electron vacancies, i.e. holes. The latter coordinates can be regarded as positions of flux quanta.

This formulation, in which electron–hole symmetry is fully explicit, leads to a simple and natural picture for the incompressible quantum Hall state at $\nu = p/q$: at some finite temperature, the density matrix is large only for configurations in which electron and hole coordinates can be grouped into clusters each containing p electron coordinates and $q - p$ hole coordinates. This electron–hole binding results dynamically from the repulsion between electrons. The creation of a neutral excitation corresponds to the breaking of such a cluster into two smaller clusters, each one leading to a local fractional-charge defect. The fact that this breaking requires a finite amount of potential energy leads to a gap in the energy spectrum and to incompressibility. The off-diagonal long-range order characterizing incompressible ground states [5] results from the fact that these clusters are bosons and they Bose condense.

The advantage of working with this finite-temperature description is that the correlation between hole coordinates, which play the role of positions of flux quanta, and electron

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coordinates can be written down explicitly. However, the price to pay is the work necessary to derive from this density matrix a microscopic description of the system at zero temperature. We show how to obtain ground-state wave functions either as a function of electron coordinates or as a function of hole coordinates in the case of the fractional quantum Hall states at $\nu = p/(2p+1)$. The point is made that the wave functions derived from this density matrix are generalizations of the pair wave functions of reference [6], which are known to be accurate. In the latter wave functions, the pairing of electrons can be regarded as resulting directly from the presence in the density matrix of clusters each containing $p = 2$ electrons and $q - p$ holes.

This description of fractional quantum Hall states based on a density matrix is in fact analogous to the one provided by the composite-fermion approach [3, 4]. Indeed, a cluster made up of one electron and one hole corresponds to a composite fermion, and a cluster made up of p electrons and $p+1$ holes, which enters into the description of the $\nu = p/(2p+1)$ state, corresponds to p composite fermions, each one belonging to a different Landau level, all bound to a flux quantum of the field experienced by composite fermions.

Let us first study the density matrix of the 2D electron gas confined to the lowest Landau level at a filling factor $\nu = p/q$ characterized by the presence of an incompressible ground state. More specifically, we consider the quantity

$$\rho_\beta(w_1, \ldots, w_{hN}, z_1, \ldots, z_{pN}) = \langle w_1, \ldots, w_{hN} | \mathrm{e}^{-\beta \tilde{V}} | z_1, \ldots, z_{pN} \rangle \tag{1}$$

where $\tilde{V}$ is the electron–electron interaction projected onto the lowest Landau level and where $|z_1, \ldots, z_{pN}\rangle$ and $|w_1, \ldots, w_{hN}\rangle$ are states respectively given in the electron representation and the hole representation, defined by

$$|z_1, \ldots, z_{pN}\rangle = c^\dagger(z_1) \cdots c^\dagger(z_{pN}) |0\rangle \tag{2}$$

$$|w_1, \ldots, w_{hN}\rangle = c(w_1) \cdots c(w_{hN}) |1\rangle \tag{3}$$

where $|0\rangle$ and $|1\rangle$ respectively denote the empty- and full-Landau-level states, $c^\dagger(z)$ is the operator creating an electron in a coherent state [7] centred at position z and h is given by $h = q - p$, and where pN and hN are respectively the number of electrons and the number of holes in the lowest Landau level.

For $\beta = 0$, the density matrix of equation (1) is simply given by

$$\rho_0 = \prod_{j_1<j_2}^{hN} (w_{j_1} - w_{j_2}) \prod_{i_1<i_2}^{pN} (z_{i_1} - z_{i_2}) \prod_{j=1}^{hN} \prod_{i=1}^{pN} (z_i - w_j) \prod_{j=1}^{hN} \mathrm{e}^{-|w_j|^2/4} \prod_{i=1}^{pN} \mathrm{e}^{-|z_i|^2/4} \tag{4}$$

in units of the magnetic length, where z_i and w_j are respectively the complex coordinates of electron i and hole j. Ignoring for a moment the distinction between the variables z_i and w_j, we see that ρ_0 has a functional form identical to that of the wave function describing a full Landau level. Thus configurations $(w_1, \ldots, w_{hN}, z_1, \ldots, z_{pN})$ characterized by non-negligible values of ρ_0 have the feature (denoted by (A)) that their coordinates are homogeneously distributed over the whole sample. Let us now study the effect of the electron–electron repulsion present in equation (1). In the case of a repulsion characterized by vanishing pseudopotential coefficients [8] for even values of the electron–electron relative angular momentum, the Hamiltonian matrix is given, up to an additive constant, by

$$-\left.\frac{\partial \rho}{\partial \beta}\right|_{\beta=0} (w_1, \ldots, w_{hN}, z_1, \ldots, z_{pN})$$

$$= -\sum_{i=1}^{pN} \sum_{j=1}^{hN} \sum_{L} \tilde{V}_L \int \mathrm{d}^2\xi \left[\mathcal{P}_{\psi_L^{(\xi)}(z_i w_j)} \rho_0 \right] (w_1, \ldots, w_{hN}, z_1, \ldots, z_{pN}) \tag{5}$$

where the summation over L is carried out over odd positive integers, the $\tilde{V}_L$ denote the pseudopotential coefficients and $\psi_L^{(\xi)}(z_1 z_2)$ is the wave function describing two particles with relative angular momentum L, and with the centre of mass in a coherent state centred at position ξ; also $\mathcal{P}$ is the projector associated with this wave function, defined by

$$\left[\mathcal{P}_{\psi_L^{(\xi)}(z_i w_j)}\rho_0\right](w_1,\ldots,w_{hN},z_1,\ldots,z_{pN}) = \psi_L^{(\xi)}(z_i w_j) \times \int \mathrm{d}^2 z_i'\ \mathrm{d}^2 w_j'\ \psi_L^{(\xi)*}(z_i' w_j')\rho_0(w_1,\ldots,w_j',\ldots,w_{hN},z_1,\ldots,z_i',\ldots,z_{pN}). \tag{6}$$

As can be seen in equation (5), the effect of the interaction at finite values of β is to increase ρ_β for configurations which have the feature (denoted by (B)) that their coordinates w_j are close to their coordinates z_i. Thus it is likely that the quantity ρ_β for small but finite β will be largest for configurations $(w_1,\ldots,w_{hN},z_1,\ldots,z_{pN})$ in the 2D plane which can be divided into compact clusters of equal sizes each containing p positions z_i and h positions w_j, which possess both feature (A) and feature (B).

Let us now construct a form for the density matrix satisfying the latter requirement for the special case of the filling fractions $\nu = p/(2p+1)$. The question of the temperature range over which this form is valid will be discussed latter. We propose to use the function given by $\kappa(w_1,\ldots,w_{hN},z_1,\ldots,z_{pN}) = \mathcal{A}_w\mathcal{A}_z\hat{\kappa}(w_1,\ldots,w_{hN},z_1,\ldots,z_{pN})$ and by

$$\hat{\kappa} = \int \prod_{n=1}^{N}\left[\mathrm{d}^2\xi_n\ \mathcal{P}_{\chi^{(\xi_n)}(z_{p(n-1)+1},\ldots,z_{pn},w_{h(n-1)+1},\ldots,w_{hn})}\right]\rho_0 \tag{7}$$

where $\mathcal{A}_w$ and $\mathcal{A}_z$ are antisymmetrization operators acting respectively on sets of variables $(w_1,\ldots,w_{hN})$ and $(z_1,\ldots,z_{pN})$ and where $\chi^{(\xi_n)}$ is a function describing the correlation of p electrons and $h = p+1$ holes in a cluster centred at position ξ_n, given by

$$\chi^{(\xi)}(z_1,\ldots,z_p,w_1,\ldots,w_h) = \prod_{i_1<i_2}^{p}(z_{i_1}-z_{i_2})\prod_{j_1<j_2}^{h}(w_{j_1}-w_{j_2})^3 \times \prod_{i=1}^{p}\exp\left(\frac{\xi^* z_i}{2}-\frac{|z_i|^2+|\xi|^2}{4}\right)\prod_{j=1}^{h}\exp\left(\frac{\xi^* w_j}{2}-\frac{|w_j|^2+|\xi|^2}{4}\right). \tag{8}$$

The projectors and the term ρ_0 in equation (7) respectively ensure that configurations leading to a large value of κ have features (B) and (A). The integrals implicit in the projectors of equation (7) can be evaluated using the identity

$$\int \prod_{i=1}^{n}\left[\mathrm{d}^2 z_i\ \exp\left(-\frac{|z_i|^2+\xi z_i^*}{2}\right)\right]P_k^*(z_1,\ldots,z_n)Q_l(z_1,\ldots,z_n)f(z_1,\ldots,z_n) = f(\xi,\ldots,\xi)\int\prod_{i=1}^{n}\left[\mathrm{d}^2 z_i\ \mathrm{e}^{-|z_i|^2/2}\right]P_k^*(z_1,\ldots,z_n)Q_l(z_1,\ldots,z_n) \tag{9}$$

where f is a polynomial, and where P_k and Q_l are homogeneous polynomials of total degree k and l with $k \leqslant l$ and are invariant under global translations of their variables [9]. This leads, up to a multiplicative constant, to

$$\hat{\kappa} = \int\prod_{n=1}^{N}\left[\mathrm{d}^2\xi_n\ \chi^{(\xi_n)}(z_{p(n-1)+1},\ldots,z_{pn},w_{h(n-1)+1},\ldots,w_{hn})\right] \times \prod_{n_1<n_2}^{N}(\xi_{n_1}-\xi_{n_2})^{q^2}\prod_{n=1}^{N}\mathrm{e}^{-q|\xi_n|^2/4}. \tag{10}$$

The function χ is chosen so as to satisfy the following criteria.

(a) $\chi^{(\xi)}$ is antisymmetric under interchange of two electron coordinates or under interchange of two hole coordinates.

(b) The total degree of the polynomial part of $\chi^{(0)}$ equals $q(q-1)/2$. Indeed, in the light of equation (9), κ would vanish for a smaller degree. On the other hand, a larger degree would lead [9] to a reduction of the power of the factors $\xi_{n_1} - \xi_{n_2}$ in equation (10), and configurations with a large value of κ would not satisfy condition (A) any longer.

(c) Factors of the type $z_i - w_j$ are absent, so χ is large for configurations satisfying condition (B).

In order to support our claim that the function κ provides a realistic description of the density matrix for some temperature, we now make the case that the ground-state wave functions derived from this form for the density matrix are generalizations of the pair wave functions of reference [6], which are known to be accurate. Let us denote by $\Phi(z_1, \ldots, z_{pN})$ and $\Theta(w_1, \ldots, w_{hN})$ the wave functions describing the ground state at $\nu = p/(2p+1)$ in the electron and hole representations of equations (2) and (3), respectively. If κ provides a good description of the density matrix, then Φ and Θ should maximize the quantity

$$R(\Phi, \Theta) = \left(\int \mathrm{d}^{2pN} Z \; \mathrm{d}^{2hN} W \; \kappa^*(W, Z) \Phi(Z) \Theta(W) \right) \times \left(\sqrt{\int \mathrm{d}^{2pN} Z \; |\Phi(Z)|^2 \int \mathrm{d}^{2hN} W \; |\Theta(W)|^2} \right)^{-1} \tag{11}$$

where $Z \equiv (z_1, \ldots, z_{pN})$ and $W \equiv (w_1, \ldots, w_{hN})$.

We now approximate $R(\Phi, \Theta)$ by substituting $\hat{\kappa}$ for κ in equation (11). Ignoring for the moment the constraint of antisymmetrization, we consider wave functions of the type

$$\hat{\Phi}(z_1, \ldots, z_{pN}) = \prod_{i=1}^{pN} \mathrm{e}^{-|z_i|^2/4} \prod_{n=1}^{N} a_{l_a}(z_{(n-1)p+1}, \ldots, z_{np}) \times \prod_{n_1<n_2}^{N} b_{l_b}(z_{(n_1-1)p+1}, \ldots, z_{n_1 p}, z_{(n_2-1)p+1}, \ldots, z_{n_2 p}) \tag{12}$$

where a_{l_a} and b_{l_b} are translationally invariant and homogeneous polynomials of total degree l_a and l_b to be determined. The polynomial

$$a_{l_a}(z_{(n-1)p+1}, \ldots, z_{np})$$

describes correlations within the group n of variables $z_{(n-1)p+1}, \ldots, z_{np}$. The polynomial

$$b_{l_b}(z_{(n_1-1)p+1}, \ldots, z_{n_1 p}, z_{(n_2-1)p+1}, \ldots, z_{n_2 p})$$

is built up using products of l_b factors of the type $(z_{(n_1-1)p+i_1} - z_{(n_2-1)p+i_2})$, with $i_1, i_2 = 1, \ldots, p$, and describes correlations between groups n_1 and n_2. We write similarly

$$\hat{\Theta}(w_1, \ldots, w_{hN}) = \prod_{j=1}^{hN} \mathrm{e}^{-|w_j|^2/4} \prod_{n=1}^{N} c_{l_c}(w_{(n-1)h+1}, \ldots, w_{nh}) \times \prod_{n_1<n_2}^{N} d_{l_d}(w_{(n_1-1)h+1}, \ldots, w_{n_1 h}, w_{(n_2-1)h+1}, \ldots, w_{n_2 h}). \tag{13}$$

We now introduce equations (10), (12) and (13) into equation (11), and integrate over coordinates z_i and w_j before integrating over coordinates ξ_n in the numerator. In order to

maximize the numerator, we demand that the results of the integration over the electron and hole coordinates have the same dependence on the variables ξ_n as the kernel of $\hat{\kappa}$ given by the last two terms of equation (10). Using equation (9), it can be seen by inspection [9] that this is possible only when l_a and l_c are equal to the degrees of the polynomial parts of equation (8), depending respectively on coordinates z_i and w_j, i.e. $l_a = p(p-1)/2$ and $l_c = 3h(h-1)/2$, and that $l_b + l_d = q^2$. The latter condition, together with the constraint that each coordinate in $\hat{\Phi}$ or in $\hat{\Theta}$ have $qN-1$ zeros, leads to $l_b = pq$ and $l_d = hq$. In the light of equations (9) and (8), it is judicious to choose

$$a_{l_a}(z_1, \ldots, z_p) = \prod_{i_1<i_2}^{p} (z_{i_1} - z_{i_2}) \tag{14}$$

$$c_{l_c}(w_1, \ldots, w_h) = \prod_{j_1<j_2}^{h} (w_{j_1} - w_{j_2})^3 \tag{15}$$

in order to obtain a large result from integration over the particle and hole coordinates in the numerator of equation (11). Among the polynomials b_{pq} and d_{hq} leading to the same value for the numerator, those yielding the most homogeneous distribution of factors $z_{i_1} - z_{i_2}$ over pairs $1 \leqslant i_1 < i_2 \leqslant pN$ and of factors $w_{j_1} - w_{j_2}$ over pairs $1 \leqslant j_1 < j_2 \leqslant hN$ lead to the smallest denominator. We therefore choose

$$b_{pq}(z_1, \ldots, z_{2p}) = \prod_{i_1=1}^{p} \prod_{i_2=1}^{p} (z_{i_1} - z_{p+i_2})^2 \sum_{P_p} \prod_{i=1}^{p} (z_{P_i} - z_{p+i}) \tag{16}$$

and

$$d_{hq}(w_1, \ldots, w_{2h}) = \prod_{j_1=1}^{h} \prod_{j_2=1}^{h} (w_{j_1} - w_{h+j_2})^2 \sum_{P_h} \prod_{j=1}^{h} (w_{P_j} - w_{h+j})^{-1} \tag{17}$$

where P_p and P_h respectively denote sums over permutations of p and h objects.

The wave functions $\Phi = \mathcal{A}_z \hat{\Phi}$ and $\Theta = \mathcal{A}_w \hat{\Theta}$ defined using equations (12), (13), (14), (15), (16) and (17) are identical to the pair wave functions of reference [6] when $p = 2$ and $h = 2$, respectively. Φ and Θ provide a good microscopic description of the ground states at filling factors $\nu = p/(2p+1)$ and $\nu = h/(2h-1)$, respectively. Indeed, they lead to numbers of flux quanta which are in agreement with the prescription of the hierarchical scheme [8] and have a large overlap with the exact ground states of the corresponding systems of particles on a sphere in the presence of Coulomb repulsion: in the case where $\nu = 2/3$, and for eight and ten particles respectively, the overlaps are 0.954 and 0.930, whereas the numbers of $L = 0$ states are two and six. In the case where $\nu = 3/5$, and for six and nine particles respectively, the overlaps are 0.988 and 0.970 and the numbers of $L = 0$ states are three and eight.

We now study the constraints imposed by statistics on the allowed values of the parameters p, h and $q = p + h$, using a path integral representation of the partition function

$$\mathcal{Z}(\beta) = \int \mathrm{d}^2 z_1 \cdots \mathrm{d}^2 z_{pN} \ \langle z_1, \ldots, z_{pN} | \mathrm{e}^{-\beta \tilde{V}} | z_1, \ldots, z_{pN} \rangle. \tag{18}$$

The exponential in equation (18) is broken into M pieces, $\exp(-\beta \tilde{V}/M)$, with M even, and the projectors

$$|z_1^{(m)}, \ldots, z_{pN}^{(m)}\rangle\langle z_1^{(m)}, \ldots, z_{pN}^{(m)}| \qquad \text{and} \qquad |w_1^{(m)}, \ldots, w_{hN}^{(m)}\rangle\langle w_1^{(m)}, \ldots, w_{hN}^{(m)}|$$

are inserted alternatively at imaginary times $m\beta/M$ for m even and m odd, respectively. We consider configurations which can be divided at all imaginary times in clusters containing

p coordinates $z_i^{(m)}$ and h coordinates $w_j^{(m)}$, which significantly contribute to the partition function. For simplicity we further demand that $z_i^{(m)}$ and $w_j^{(m)}$ have a smooth dependence on m, and belong to the same cluster throughout imaginary time. The phase corresponding to a given path is a product of phase contributions associated with each time step, which can in turn be evaluated by means of equation (4). Ignoring the factors $z_i - w_j$ for z_i and w_j belonging to the same cluster, this leads to $(-1)^{ph[P]}$, where $[P]$ is the parity of the permutation taking place among the clusters between the imaginary time $\tau = 0$ and $\tau = \beta$. Thus such paths contribute constructively to the partition function only if ph is even—that is, if (a) p is even and h is odd or (b) p is odd and h is even or (c) both p and h are even. Note that cases (a) and (b) imply a filling factor of $\nu = p/q$ with an odd denominator.

The fact that all of the incompressible fractional quantum Hall states observed in the lowest Landau level are characterized by a filling factor with an odd denominator may be attributed to the fact that a stable cluster made up of p electron coordinates and h hole coordinates can only be obtained if one of the parameters is odd, thus corresponding to case (a) or case (b). Indeed, for p and q both even, the paths obtained by clustering p coordinates z_i and h coordinates w_j could be energetically unfavourable compared with paths obtained by clustering $p/2$ coordinates z_i and $h/2$ coordinates w_j.

The incompressible state at $\nu = 5/2$ is characterized by $p = h = 2$ (case (c)). The absence of an incompressible state at $\nu = 1/2$ is attributed [10] to the fact that, in the lowest Landau level, clusters with $p = h = 2$ are unstable toward the formation of clusters with $p = h = 1$. Since the paths of such $p = h = 1$ clusters contribute to the partition function with a phase $(-1)^{[P]}$ reminiscent of that of fermions, they can be regarded as composite fermions [4].

It is important to note that the approximation to the density matrix provided by the function κ of equation (10) is unable to describe neutral excitations consisting in pairs of well-separated fractionally charged excitations. Indeed, when using this approximate form of the density matrix to evaluate the number of electrons in a surface large compared to the squared magnetic length, one obtains fluctuations much too small to include the effect of such excitations. In order to include the effect of a single quasiparticle–quasihole pair, it is necessary to add to κ a further term. This additional term can be obtained from the right-hand side of equation (7) by replacing one of the projectors associated with clusters of p electrons and h holes by two projectors: one associated with a cluster of p' electrons and h' holes and another associated with a cluster of $p - p'$ electrons and $h - h'$ holes, with p' and h' satisfying $p'h - ph' = \pm 1$. The cluster associated with the plus sign leads to a local charge defect e/q (a quasiparticle) and the other to $-e/q$ (quasihole), where e is the electron's charge. The parameters p' and h' are given by

$$p'/h' = 1/(n_1 + 1/(n_2 + \cdots + 1/n_{k-1}))\cdots))$$

in terms of the continued-fraction decomposition

$$p/h = 1/(n_1 + 1/(n_2 + \cdots + 1/n_k))\cdots)).$$

Thus the creation of a neutral excitation corresponds to the breaking of a cluster of p electrons and h holes. The fact that this breaking requires a finite amount of potential energy, together with the fact that κ from equation (7) considered as a function of $z_1, \ldots, z_{pN}$ or of $w_1, \ldots, w_{hN}$ has a small overlap with the contribution described above including a neutral excitation, leads to a gap in the energy spectrum and to incompressibility.

We finally address the question of the temperature range in which the function $\kappa(w_1, \ldots, w_{hN}, z_1, \ldots, z_{pN})$ of equation (7) may be adequate for describing the density matrix of equation (1). Although κ is unable to describe neutral excitations consisting in

pairs of well-separated elementary excitations which are associated with short-wavelength collective modes [11], we do believe that the effects of long-wavelength collective modes are included in $\kappa(w_1, \ldots, w_{hN}, z_1, \ldots, z_{pN})$ and are responsible for the correlations existing between the variables z_i and w_j. These collective modes are characterized by a finite excitation energy $\epsilon_{k=0}$. We thus conjecture that the temperature for which κ is adequate for describing the density matrix is of the order of $\epsilon_{k=0}/k_B$.

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References

[1] Laughlin R B 1983 *Phys. Rev. Lett.* **50** 1395
[2] Read N 1989 *Phys. Rev. Lett.* **62** 86
[3] Jain J K 1989 *Phys. Rev. Lett.* **63** 1223
[4] Halperin B I, Lee P A and Read N 1993 *Phys. Rev.* B **47** 7312
[5] Girvin S M and MacDonald A H 1987 *Phys. Rev. Lett.* **58** 1252
[6] Morf R, d'Ambrumenil N and Halperin B I 1986 *Phys. Rev.* B **34** 3037
[7] Girvin S M and Jach T 1984 *Phys. Rev.* **29** 5617
[8] Haldane F D M 1987 *The Quantum Hall Effect* ed R E Prange and S M Girvin (Berlin: Springer)
[9] This identity can be established using a transformation to centre-of-mass and relative coordinates. If $k > l$, the left-hand side of equation (9) is proportional to a linear combination of terms of the type $\partial_{z_1}^{l_1} \cdots \partial_{z_n}^{l_n} f|_{z_1=\cdots=z_n=\xi}$ with $l_1 + \cdots + l_n = k - l$.
[10] Ho T L 1995 *Phys. Rev. Lett.* **75** 1186
[11] Laughlin R B 1987 *The Quantum Hall Effect* ed R E Prange and S M Girvin (Berlin: Springer)